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DISTORTION OF TELLURIC FIELD MEASUREMENTS NEAR HILLS USING A CONFORMAL MAPPING METHOD

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Abstract

The effect of a two dimensional hill surface on the telluric field measured along the perpendicular direction has been studied near a horizontal surface using Schwarz-Christoffel conformal transformation technique. Though the solution presented here is valid for various model parameters of the hill, this paper is restricted to consideration of symmetric surfaces. The effect has been analysed by varying the normalised height from the surface and its inclination with respect to the horizontal surface. The results indicate that both the height of the hill and its inclination affect telluric field measurements significantly. For example, to obtain telluric field measurements within 10 percent of their undisturbed values for a basement depth 1 km. and the height of the hill 500 m model, the station should be located more than 60 m from an 80° inclined surface and more than about 150 m for a 20° inclined surface.

Introduction

Telluric and magnetotelluric methods have an advantage over other geoelectrical methods because of their deep level of investigation at low cost. However, just like any other geophysical methods the observed data need correction due to near surface inhomogeneities, surface topographic effects etc. To avoid topographic effects, the station locations should ideally be located in relatively flat areas. Modelling algorithms, in general, assumes a horizontal surface, however, where the Earth's surface is non-horizontal the data should be suitably corrected so that the assumption is valid. In recent years, with the advent of fast computers and the development of rigorous numerical algorithms it is possible to accommodate non-horizontal topographic earth surfaces in the modelling schemes (Ramaswamy et al, 1976; Wannamaker et al, 1986; Chouteau and Bouchard, 1988; Jiracek et al, 1989 etc.). Apart from numerical studies, topographic problems have been examined using analog models (Wescott and Hessler, 1962; Faradzhev et al, 1972; etc.) and using conformal mapping techniques (Kunetz and de Gery, 1956; Thayer, 1975; Harinarayana and Sarma, 1982). Analytical methods have an advantage over numerical methods because of their capability of analysing the variation of different parameters of the model with less computation.

In the present study a two-dimensional topographic hill model (fig.1) is considered and the distortion effects on telluric field measurements perpendicular to

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the model are determined using Schwarz-Christoffel conformal transformation technique (Walker, 1964). The model assumed in the present study is bounded by the air above and a high resistive basement below. This assumption is not unreasonable. For example, the electrical basement is usually several orders of magnitude more resistive than overlying sedimentary rocks.

To solve the problem of the hill geometry, the assumed model (fig.1) in z-plane is transferred to another, w-plane, using the Schwarz-Christoffel transformation. The long period (DC) telluric field is computed in the w-plane and then transferred back to the z-plane. The solution of the present problem can be treated as semi-analytical in the sense that the relevant differential equation is solved with numerical integration using the Runge-Kutta method. A brief description of the theory is given here.

Theory

Formulation of the problem:

The period of telluric field signals considered here is sufficiently long and can be approximated by a DC field, for which the potential satisfies Laplace's equation.

$$\nabla^2 V = 0$$

Where V is the potential. Along the boundary EDCBAA'E' in fig. 1, the normal gradient vanishes, i.e.,

$$\frac{\partial V}{\partial n} = 0$$

By conformal transformation, the complex geometry in the z-plane can be mapped into the w-plane to form a simple geometry with the same boundary conditions, the potential still satisfying Laplace's equation. The Schwarz-Christoffel transformation of the geometry of the structure can be written in differential form as

$$\frac{dz}{dw} = p w^{\frac{\alpha}{\pi}} (w-k)^{\frac{-(\alpha+\beta)}{\pi}} (w-l)^{\frac{\beta}{\pi}} (w-1)^{-1} \qquad \text{.........(1)}$$

where, α and β are the angles of the inclined surface of the hill. The constants p,k and l can be determined by integrating along EE' and AA'. Therefore,

$$p = \frac{H_1}{\pi} \qquad \text{...............(2)}$$

where, H_1 is the depth to the basement from the horizontal surface and

$$k = 1 - [(1-l)^{\frac{\beta}{\pi}}]^{\frac{\pi}{\alpha+\beta}} \qquad \text{...............(3)}$$

From the above equation it is clear that the values k and l cannot be obtained uniquely. However, the value of l can be assumed such that $0 < l < 1$. As the object of this study is to estimate the effect of topography at points on the horizontal surface, it is necessary to map the x-axis in the z-plane onto the u-axis in w-plane. To solve this, the procedure used in Roy and Naidu (1972) can be adopted as follows.

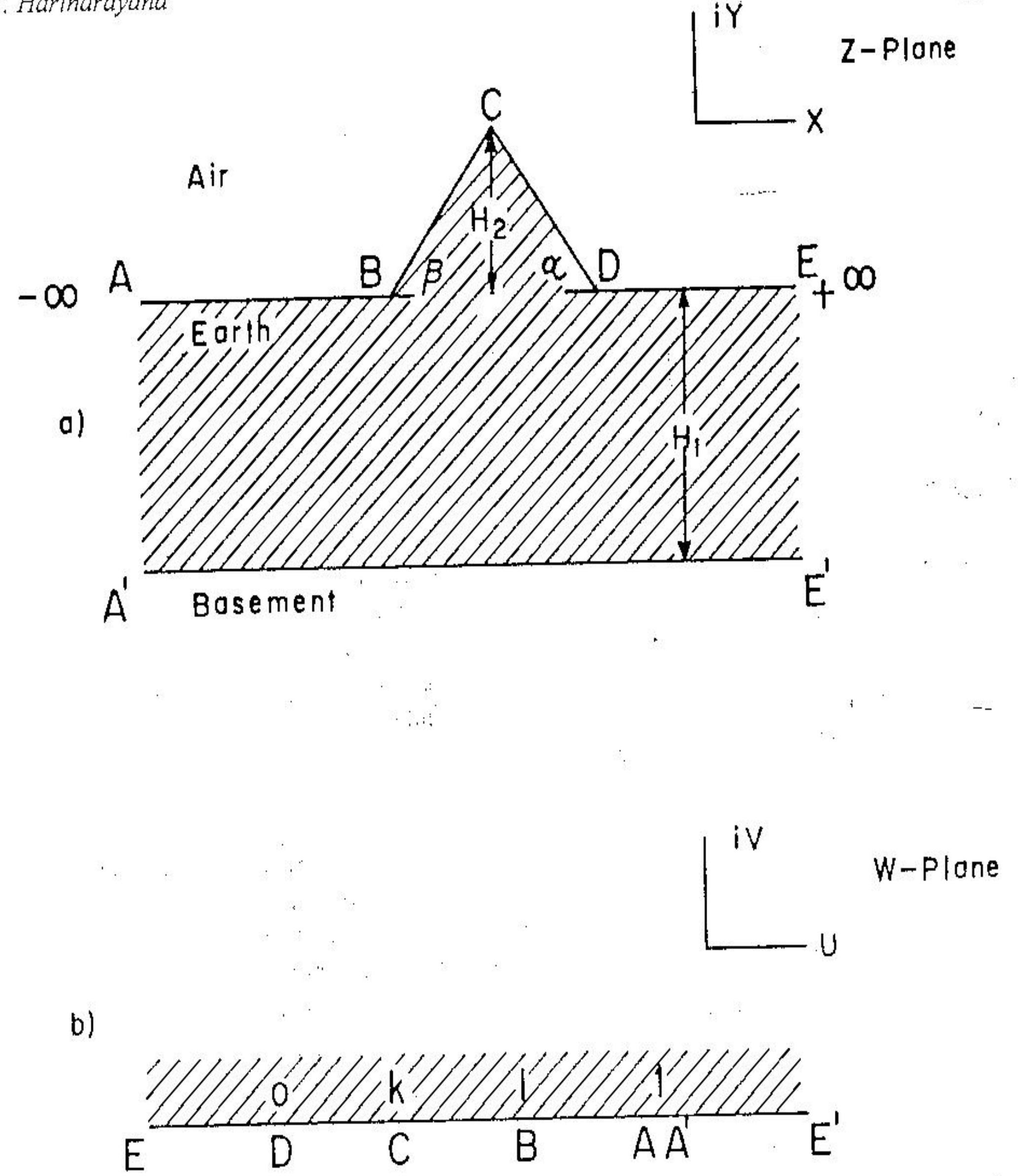

Fig.1(a):Geometry of the hill surface model in z-plane. The shaded region represents the conducting region of the earth. Air and electrical basement resistivities are assumed infinite. The surface BCD represents the hill surface and A'E' is the electrical basement surface. b) Schwarz-Christoffel transformation of the surface shown in (a).

Equation 1 can be written in real and imaginary parts as,

$$\frac{\partial u}{\partial x} = \text{Real}\left[\frac{1}{p}\; w^{\frac{-\alpha}{\pi}}\,(w-k)^{\frac{\alpha+\beta}{\pi}}\,(w-l)^{\frac{-\beta}{\pi}}\,(w-1)\right] \qquad \text{..............(4)}$$

$$\frac{\partial v}{\partial x} = \text{Imag}\left[\frac{1}{p}\; w^{\frac{-\alpha}{\pi}}\,(w-k)^{\frac{\alpha+\beta}{\pi}}\,(w-l)^{\frac{-\beta}{\pi}}\,(w-1)\right] \qquad \text{..............(5)}$$

Since equations 4 and 5 are nonlinear

differential equations, they can be solved numerically, integrating simultaneously along D to B keeping y=0 in z-plane. However, to start the integration one requires initial conditions. At the point D the above equations become singular, because u=v=0. This difficulty can be overcome by considering the asymptotic solution of equation 1 (for details see Naidu, 1965).

Therefore,

$$\frac{v}{u} = \tan\left[\frac{\alpha}{1+\frac{\alpha}{\pi}}\right] \qquad \text{...............(6)}$$

Either v or u may be assumed << 1. Suppose u << 1, then the initial conditions can be written as,

$$v_0 = u_0 \tan\left[\frac{\alpha}{1+\frac{\alpha}{\pi}}\right] \qquad \text{...............(7)}$$

and

$$x_0 = \left[\frac{-pk^{\frac{-(\alpha+\beta)}{\pi}}\, l^{\frac{\beta}{\pi}}}{1+\frac{\alpha}{\pi}}\right](u^2+v^2)^{\frac{1+\frac{\alpha}{\pi}}{2}} \qquad \text{...............(8)}$$

Using these initial conditions it is now possible to integrate equations 4 and 5.

After integration, u approaches l at the point B, v approaches 0 and x assumes a certain value which inturn depends on the assumed value of l. Knowing the values of x, α and β, the height of the hill H_2 can be determined. Since v=0 along BA, the equation required for mapping x-axis on the u-axis can be written as,

$$\frac{\partial u}{\partial x} = \left[\frac{1}{p}\, u^{\frac{-\alpha}{\pi}}\,(u-k)^{\frac{\alpha+\beta}{\pi}}\,(u-l)^{\frac{-\beta}{\pi}}(u-1)\right] \qquad \text{..............(9)}$$

This may be considered as a non linear differential equation relating u in w-plane and x in z-plane and can be solved again numerically using the values obtained after solving equations 4 and 5. After transforming the complex geometry and finding the solution for mapping x-axis onto the u-axis, the remaining problem is the computation of the telluric field.

Computation of telluric field :

The telluric field in the z-plane can be considered as if produced from a line source and sink placed at + and − respectively. The potential distribution in w-plane for such a situation is given by

$$V = \frac{I/\pi}{\operatorname{Log}(w-1)} \qquad \text{.............(10)}$$

where, I is the current strength.

Differentiation of this potential will give the telluric field E in w-plane in the form,

$$E = -\frac{dV}{dw} = \frac{I}{\pi}\cdot\frac{1}{w-1} \qquad \text{...............(11)}$$

Therefore, transforming this field to the z-

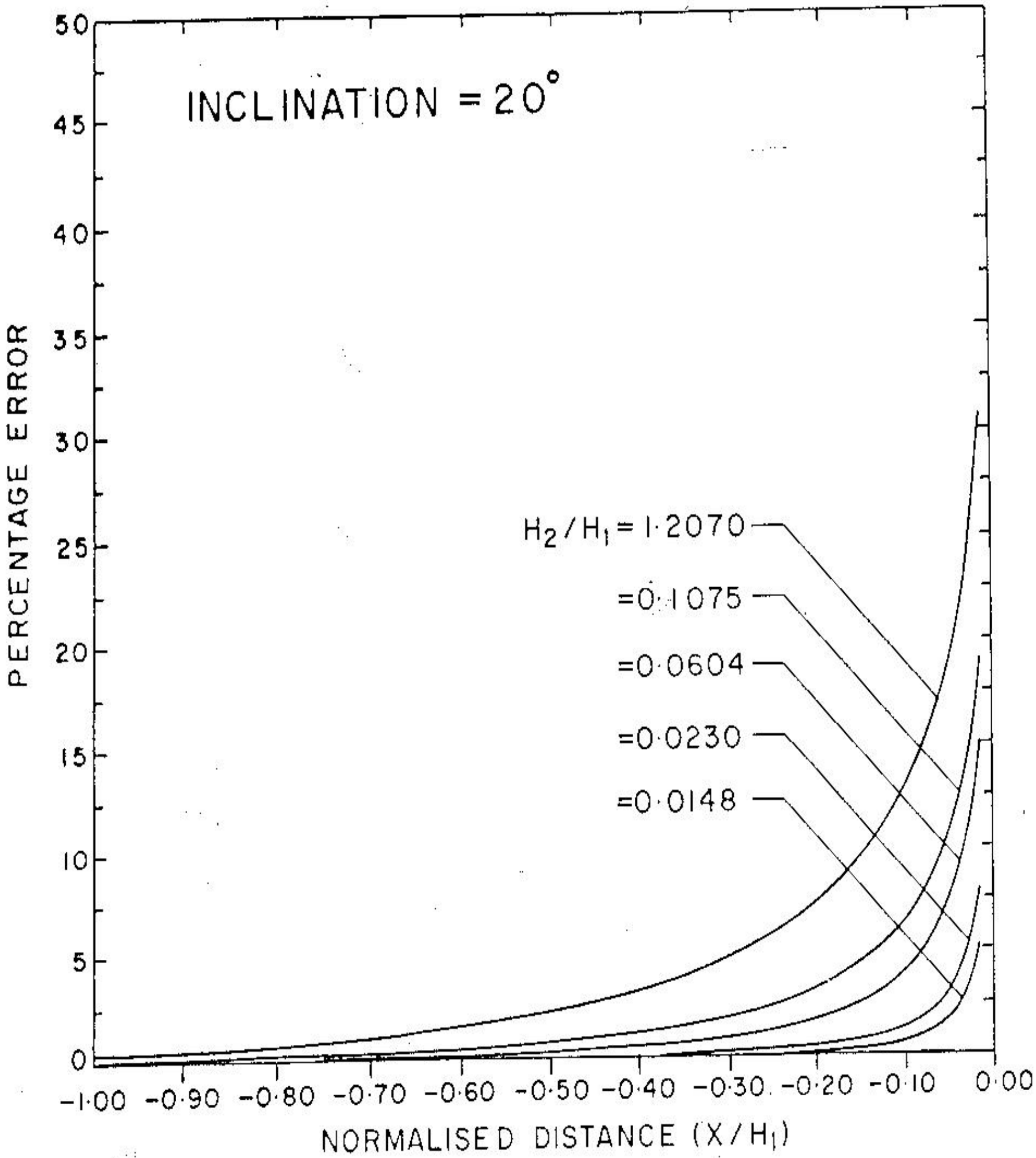

Fig.2: Distortion of telluric field measurements shown as percentage error near a hill with varying normalised height (H_2/H_1) parameter. Percentage error computed with respect to the undisturbed value observed far away from the hill surface. The inclination ($\alpha = \beta$) is assumed to be 20°. Distance x is normalised with H_1.

plane, we have,

$$E = -\frac{dv}{dz} = \left[\frac{I}{H_1} w^{\frac{-\alpha}{\pi}} (w-k)^{\frac{\alpha+\beta}{\pi}} (w-l)^{\frac{-\beta}{\pi}}\right], \quad \text{..............(12)}$$

Since, along BA v = 0 we have,

$$E = -\frac{dv}{dx} = \left[\frac{I}{H_1} u^{\frac{-\alpha}{\pi}} (u-k)^{\frac{\alpha+\beta}{\pi}} (u-l)^{\frac{-\beta}{\pi}}\right] \quad \text{...............(13)}$$

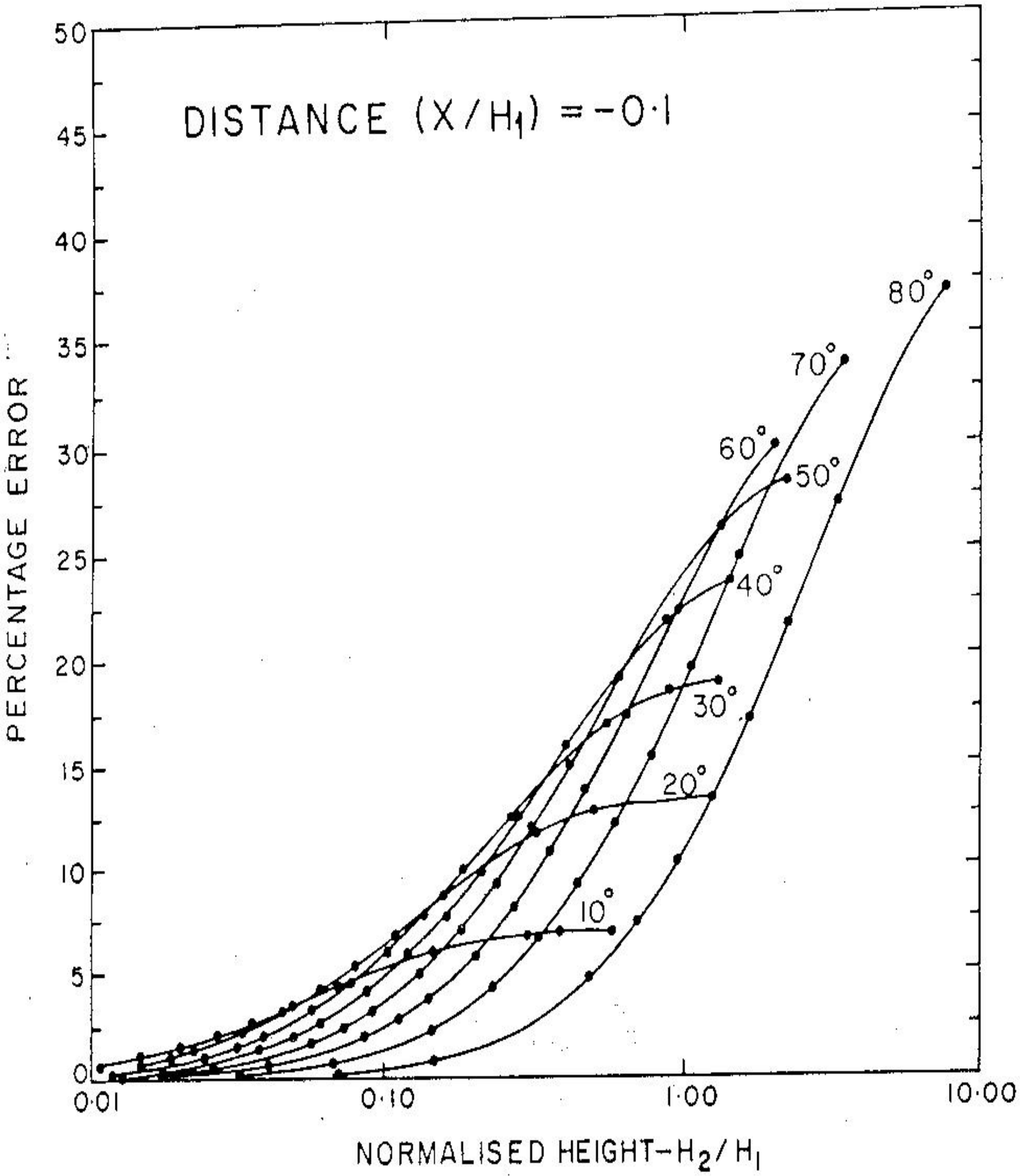

Fig.3: Effect of topographic hill surface observed at a distance, x/H_1 = —0.1. The curves shown are for different values of inclination.

Discussion

The solution derived can be studied by varying six parameters: the angles α and β of the hill surface, the elevation H_2 , the depth to the basement H_1 ,the distance x from the offset (i.e. along BA in fig.1) and the distortion of the telluric field. Thus one can have many families of computed curves. However, in the present study it is assumed that $\alpha = \beta$, i.e., restricted to a symmetric model and that the parameters H_1 ,H_2 and x are normalised. The distortion of the telluric field is represented as a percentage error computed with respect to the value observed at a large distance x. As a result, the number

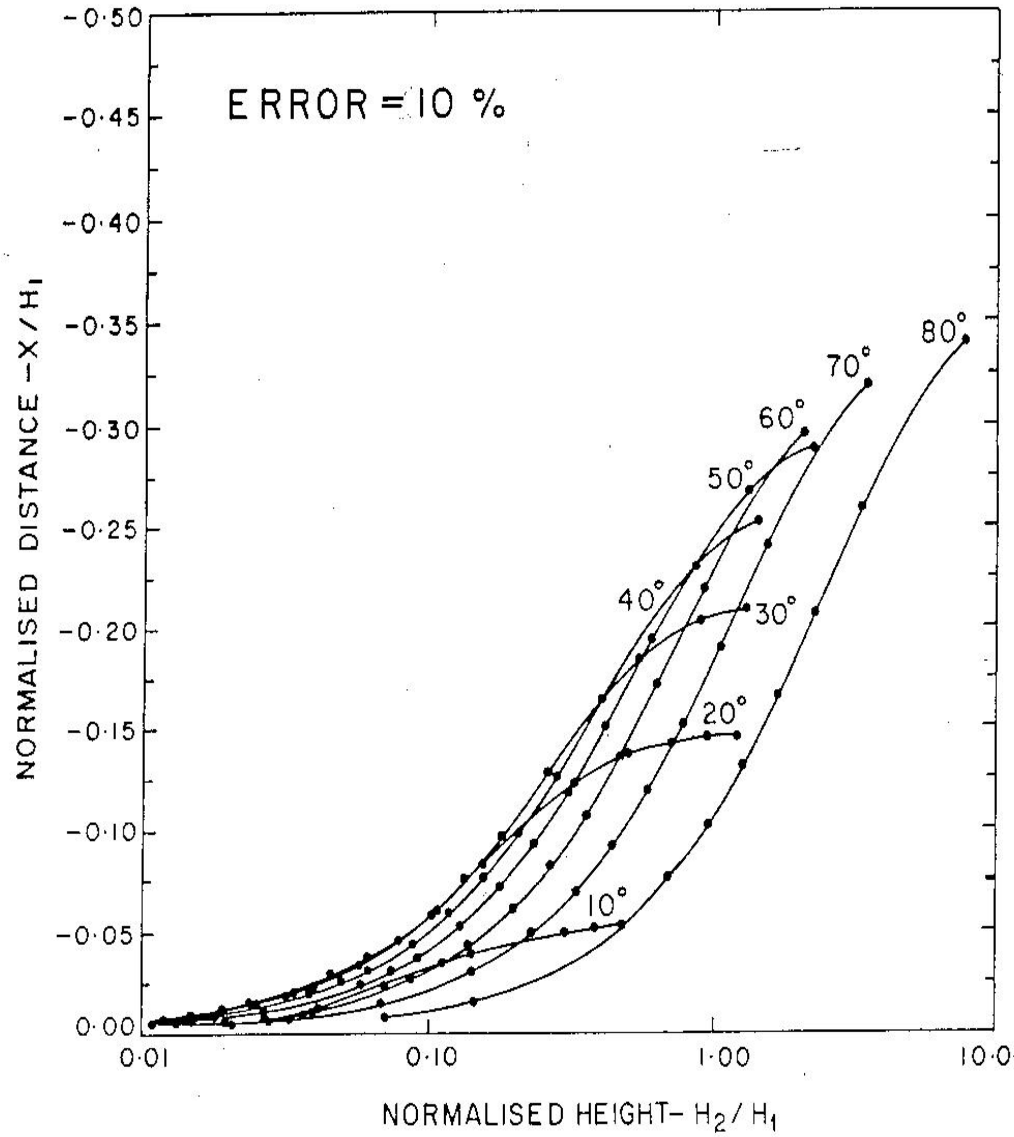

Fig.4: Normalised distance (x/H_1) as a function of normalised height (H_2/H_1). The percentage error is assumed to be invariant and the curves shown are for different values of inclination.

of variables reduces to four: inclination, normalised distance, normalised elevation and the percentage error, making the analysis of the results of the study more tractable. Among the four parameters, each one is assumed invariant in turn, two of the parameters are specified on the x and y axes and curves representing the fourth parameter are plotted as shown in Figures 2 to 4.

With the above convention, the distortion of telluric field measurements due to topography is studied first with the inclination ($\alpha = \beta$) assumed invariant and varying the normalised depth parameter H_2/H_1 (fig.2). It can be seen that if we study the error at a particular distance (say $x / H_1 = -0.2$), the error increases as the values of H_2/H_1 increases and also more elevated hill distorts the measurements to a larger distance. The effect of the inclination observed at a constant distance from the offset is shown in fig.3. From this figure one can observe that for any value of α, as the H_2/H_1 increases the error also increases, however for larger values of H_2/H_1 all the curves show asymptotic behaviour i.e. error remains constant for very large values of H_2/H_1 .It can also be seen that for lower values of H_2/H_1 ,the error increases as α decreases. This can be seen by considering the following example. At a normalised distance, $x/H_1 = -0.1$, if $H_2/H_1=1.0$. the error due to topography is about 10% for $\alpha = 80^0$ and it is nearly 25 % for $\alpha = 50^0$. This is not unreasonable, because in the present model, for the same height of the hill, a small inclined surface model has a large area of cross section of hill surface (due to large value of BD in fig.1a) compared with steep hill model.

fig.4 shows the normalised distance necessary in order to limit the distortion of telluric field measurements to within 10% of their undistorted value. The curves are plotted in the figure for different α values. It can be seen that as H_2/H_1 increases, the normalised distance (x/H_1) necessary to limit the error within 10% also increases. The curves, however, show an asymptotic trend to a value of x/H_1 for larger H_2/H_1 values. Again for the same reason discussed earlier for the results presented in fig.3, it can be verified that for a lower value of H_2/H_1 ,a model with smaller α value (i.e. less inclined topography) distorts the measurements to larger distances than does a model with larger α values (i.e. more inclined topography).

Conclusion

An attempt has been made in the present study to analyse the topographic effect of a hill on telluric field measurements by varying different parameters. From the models considered in the present study it is observed that the topographic effect is observed to increase with the height of the hill. For a particular distance, from the hill it is observed that as the height of the hill increases, the distortion effect due to topography also increases but it soon reaches a maximum and remains constant irrespective of height of the topography. Nomograms are presented which may help the exploration Geophysicist in formulating a rule of thumb to locate field stations to minimise the distortions of telluric field measurements due to topography.

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